

APPROXIMATIONS

(For Board)

Approximation by Differentials :

A method for **approximating** the value of a **function** near a known value.

The method uses the **tangent line** at the known value of the function

to **approximate** the function's **graph**.

In this method, Δx and Δy represent the changes in x and y for the **function**,

and dx and dy represent the changes in x and y for the **tangent line**.

If $y = f(x)$ and Δx is a **small change** in x then the corresponding change in y (**approximately**) is given by $f'(x)\Delta x$.

This is called the **differential** of y and is denoted by dy

$$\therefore dy = f'(x) \Delta x$$

The actual change or the actual error in y is denoted by

$$\Delta y = f(x + \Delta x) - f(x)$$

$$\Delta y \cong dy = f'(x) \Delta x$$

Geometrical Meaning of approximations

Let $f : D \rightarrow \mathbb{R}$, $D \subset \mathbb{R}$, be a given function and let $y = f(x)$.

Let Δx denote a **small increment** in x .

The increment in y corresponding to the increment in x ,

$$\Delta y = f(x + \Delta x) - f(x)$$

denoted by Δy

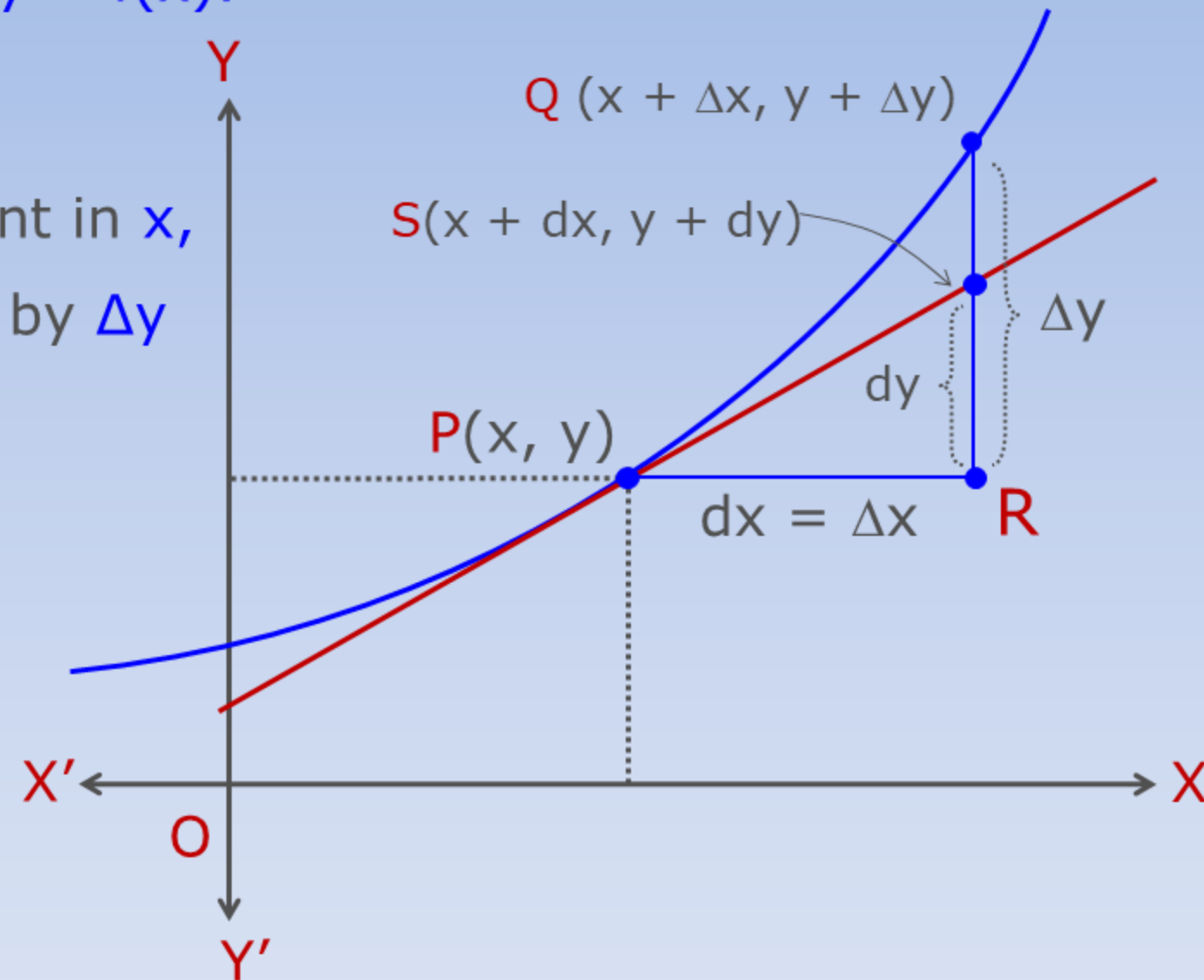
(i) The differential of x , denoted by dx ,
is defined by $dx = \Delta x$.

(ii) The differential of y , denoted by dy ,

is defined by $dy = f'(x)dx$ or $dy = \left(\frac{dy}{dx}\right) \Delta x$

In case $dx = \Delta x$ is relatively small when compared with x ,

dy is a good approximation of Δy and we denote it by $dy \approx \Delta y$.



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Use differential to approximate $\sqrt{36.6}$.

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Use differential to approximate $\sqrt{36.6}$.

$$y = \sqrt{x} \quad \text{Let } x = 36 \text{ \& } \Delta x = 0.6$$

$$\Delta y = f(x + \Delta x) - f(x)$$

$$\Delta y = \sqrt{x + \Delta x} - \sqrt{x} = \sqrt{36.6} - \sqrt{36}$$

$$= \sqrt{36.6} - 6$$

$$\Rightarrow \sqrt{36.6} = 6 + \Delta y$$

$dy \approx \Delta y$ and is given by $dy = \left(\frac{dy}{dx}\right)\Delta x$

$$dy = \frac{1}{2\sqrt{x}} (0.6) = \frac{1}{2 \times 6} (0.6) = 0.05$$

$$y = \sqrt{x} \Rightarrow y = x^{1/2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} x^{1/2} = \frac{1}{2} x^{1/2 - 1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$$

Thus, the **approximate value** of $\sqrt{36.6} = 6 + \Delta y = 6 + 0.05 = 6.05$.

Use differential to approximate $(25)^{1/3}$.

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Use differential to approximate $(25)^{1/3}$.

$$y = x^{1/3} \quad \text{Let } x = 27 \text{ \& } \Delta x = -2$$

$$\Delta y = f(x + \Delta x) - f(x)$$

$$\begin{aligned} \Delta y &= (x + \Delta x)^{1/3} - x^{1/3} = (25)^{1/3} - (27)^{1/3} \\ &= (25)^{1/3} - 3 \end{aligned}$$

$$\Rightarrow (25)^{1/3} = 3 + \Delta y$$

$dy \approx \Delta y$ and is given by $dy = \left(\frac{dy}{dx}\right)\Delta x$

$$dy = \frac{1}{3x^{2/3}}(-2) = \frac{1}{3 \times 27^{2/3}}(-2) = \frac{-2}{27} = -0.074$$

$$\begin{aligned} y = x^{1/3} &\Rightarrow \frac{dy}{dx} = \frac{d}{dx} x^{1/3} \\ &= \frac{1}{3} x^{1/3 - 1} \\ &\Rightarrow \frac{dy}{dx} = \frac{1}{3} x^{-2/3} = \frac{1}{3x^{2/3}} \end{aligned}$$

Thus, the **approximate value** of $(25)^{1/3} = 3 + \Delta y = 3 - 0.074 = 2.926$.

Find the approximate value of $f(3.02)$, where $f(x) = 3x^2 + 5x + 3$.

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Find the approximate value of $f(3.02)$, where $f(x) = 3x^2 + 5x + 3$.

Let $x = 3$ and $\Delta x = 0.02$.

$$y = 3x^2 + 5x + 3$$

$$\frac{dy}{dx} = 3 \frac{d}{dx}x^2 + 5 \frac{d}{dx}x + \frac{d}{dx}3 = 3(2x) + 5(1) + 0 = 6x + 5$$

$dy \approx \Delta y$ and is given by $dy = \left(\frac{dy}{dx}\right)\Delta x$

$$\Delta y = f(x + \Delta x) - f(x)$$

$$\Rightarrow f(x + \Delta x) = \Delta y + f(x)$$

$$\Rightarrow f(x + \Delta x) = \left(\frac{dy}{dx}\right)\Delta x + f(x)$$

$$f(x + \Delta x) = (6x + 5)(0.02) + 3x^2 + 5x + 3$$

$$f(3 + 0.02) = (6 \times 3 + 5)(0.02) + 3(3)^2 + 5 \times 3 + 3$$

$$\Rightarrow f(3.02) = (23)(0.02) + 3(9) + 15 + 3$$

$$= 45.46$$

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Find the approximate change in the volume V of a cube of side x meters caused
by increasing the side by 2%.

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Find the approximate change in the volume V of a cube of side x meters caused
by increasing the side by 2%.

Let **side** of the **cube** = x metres

Increase in size = 2% = $0.02x$

$$\Delta x = 0.02x$$

Volume of a cube $V = x^3$

$$\frac{dV}{dx} = \frac{d}{dx}x^3 = 3x^2$$

Approximate change in volume is ΔV

$$\begin{aligned}\Delta V &= \left(\frac{dV}{dx}\right) \Delta x = (3x^2) \Delta x \\ &= (3x^2) (0.02x) = 0.06x^3 \text{ m}^3\end{aligned}$$

If the radius of a sphere is measured as 9 cm with an error of 0.03cm, then find the approximate error in calculating its volume.

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If the radius of a sphere is measured as 9 cm with an error of 0.03cm, then find the approximate error in calculating its volume.

Let r be the radius of the sphere and

Δr be the error in measuring the radius.

$$r = 9 \text{ cm} \quad \text{and} \quad \Delta r = 0.03 \text{ cm.}$$

The volume of the sphere is $V = \frac{4}{3} \pi r^3$

$$\Rightarrow \frac{dV}{dr} = \frac{4}{3} \pi \frac{d}{dr} r^3 = \frac{4}{3} \pi 3r^2$$

$$\Rightarrow \frac{dV}{dr} = 4\pi r^2$$

The approximate error in the volume is ΔV

$$\begin{aligned} \therefore \Delta V &= \left(\frac{dV}{dr} \right) \Delta r = (4\pi r^2) \Delta r \\ &= 4\pi(9)^2 (0.03) = 9.72\pi \text{ cm}^3 \end{aligned}$$

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